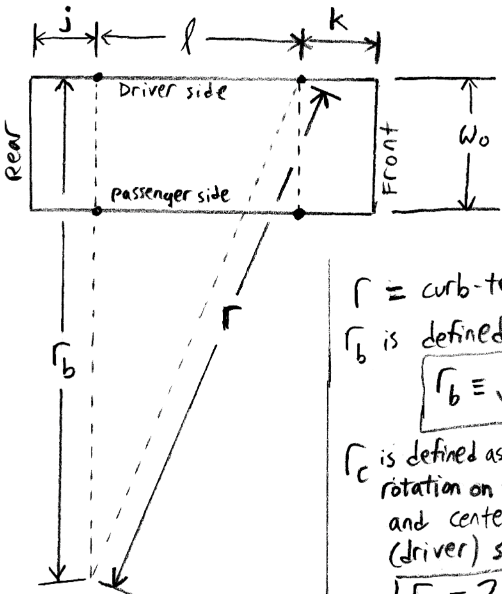
# Parallel Parking Optimization, Pt. 1

What is the minimal space needed for parallel parking?




$l$ = wheel base

$k$ = front axle to front bumper

$j$ = rear axle to rear bumper

$w_0$ = width of car being parked

$r$ = curb-to-curb turn radius

$r_b$ is defined as shown such that

$$\boxed{r_b \equiv \sqrt{r^2 - l^2}}$$

$r_c$ is defined as distance between center of rotation on right (passenger) side and center of rotation on left (driver) side such that

$$\boxed{r_c \equiv 2r_b - w_0}$$

$m$ is defined as shown such that

$$\boxed{m \equiv r_b - w_0 - d_c + w_1}$$

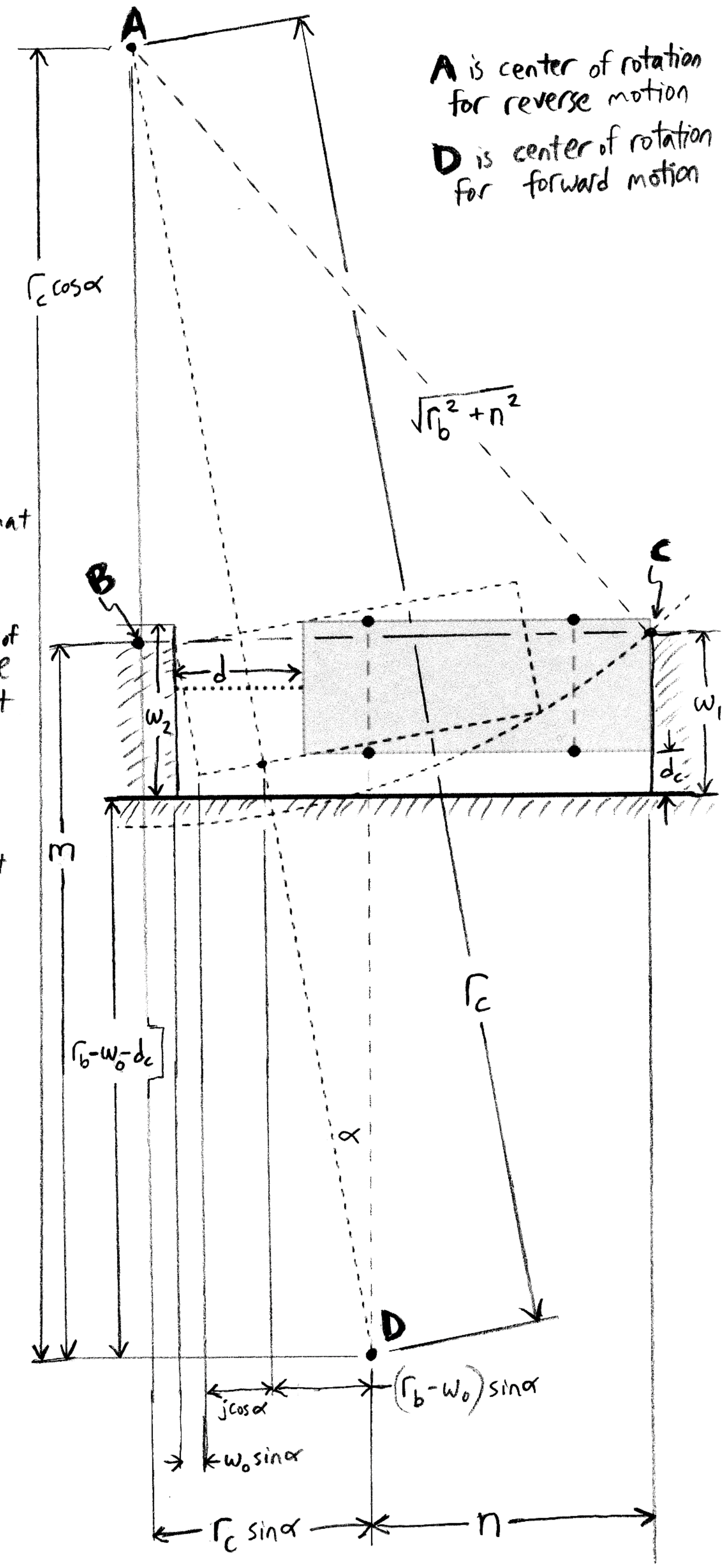
$n$ is defined as shown such that

$$\boxed{n \equiv l + k}$$

$d_c$ = distance from curb when parked

$w_1$ = distance that parked car in front protrudes from curb (car width plus space from curb)

$w_2$ = distance that parked car behind protrudes from curb (assumptions make this dimension irrelevant)




## Variables to be solved:

$\alpha$ = acute angle that side of car forms with curb at the moment that it stops reversing and starts moving forward

$d$ = minimum excess length of parking space required beyond one full car length

Assumptions:

1. Optimized parking motion occurs as follows: At some "ideal" start position, steering wheel is turned hard away from curb, car backs up along an arc, car stops, steering wheel is turned hard toward curb, and car pulls forward along an arc until parallel to curb.

2. Desired distance from curb, $d_c$, will be achieved in one iteration of reverse-forward motion. In actuality, closer distance to curb may be achieved by successive reverse-forward iterations.

3. Optimized geometry (i.e. minimal value of $d$) occurs when, after pulling forward until parallel to curb, there is no distance between front bumper and parked car in front.

4. When backing up, collision with behind parked car would occur as contact between rear-left (driver side) corner of moving car and some point along the front bumper of the parked car.
   Separate equations would be required for the alternate case of some point along the rear bumper of moving car contacting front-left corner of parked car.

5. the axis of rotation for the moving car intersects the line on which the rear axle lies.

6. Curb-to-curb radius $r$ is the distance between the moving car's axis of rotation and the center of the outer-front tire's outer wall.

7. The distance from the curb, $d_c$, must be selected such that the moving car does not run up onto the curb during the reverse motion. The following inequality, if satisfied, verifies that the center of the rear-right (passenger) tire does not cross the curb line:

$$(r_b - w_o)\cos\alpha \geq r_b - w_o - d_c$$

$$d_c \geq (1-\cos\alpha)(r_b - w_o)$$

8. The owners of the parked cars in front and behind won't mind you gently playing "bumper cars" with their vehicles.

## Derivation of formulas for $\alpha$ and $d$

Pythagorean Theorem on $\Delta ABC$ yields

$$(AB)^2 + (BC)^2 = (AC)^2$$

$$(r_c \cos\alpha - m)^2 + (r_c \sin\alpha + n)^2 = \left(\sqrt{r_b^2 + n^2}\right)^2$$

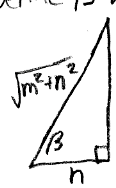
$$r_c^2 \cos^2\alpha - 2 r_c m \cos\alpha + m^2 + r_c^2 \sin^2\alpha + 2 r_c n \sin\alpha + n^2 = r_b^2 + n^2$$

$$r_c^2 + 2 r_c (n \sin\alpha - m \cos\alpha) + m^2 = r_b^2$$

$$(\sin\alpha)(n) - (\cos\alpha)(m) = \frac{r_b^2 - r_c^2 - m^2}{2 r_c}$$

$$(\sin\alpha)\left(\frac{n}{\sqrt{m^2+n^2}}\right) - (\cos\alpha)\left(\frac{m}{\sqrt{m^2+n^2}}\right) = \frac{r_b^2 - r_c^2 - m^2}{2 r_c \sqrt{m^2+n^2}}$$

Define $\beta$ as such:




$\beta = \tan^{-1}\left(\frac{m}{n}\right)$

$$\sin\alpha \cos\beta - \cos\alpha \sin\beta = \text{''}$$

$$\sin(\alpha - \beta) = \text{''}$$

$$\alpha - \beta = \sin^{-1}\left(\frac{r_b^2 - r_c^2 - m^2}{2 r_c \sqrt{m^2+n^2}}\right)$$

$$\boxed{\alpha = \sin^{-1}\left(\frac{r_b^2 - r_c^2 - m^2}{2 r_c \sqrt{m^2+n^2}}\right) + \tan^{-1}\left(\frac{m}{n}\right)}$$

$$d = (r_b - w_o)\sin\alpha + j\cos\alpha + w_o \sin\alpha - j$$

$$\boxed{d = r_b \sin\alpha - j(1 - \cos\alpha)}$$

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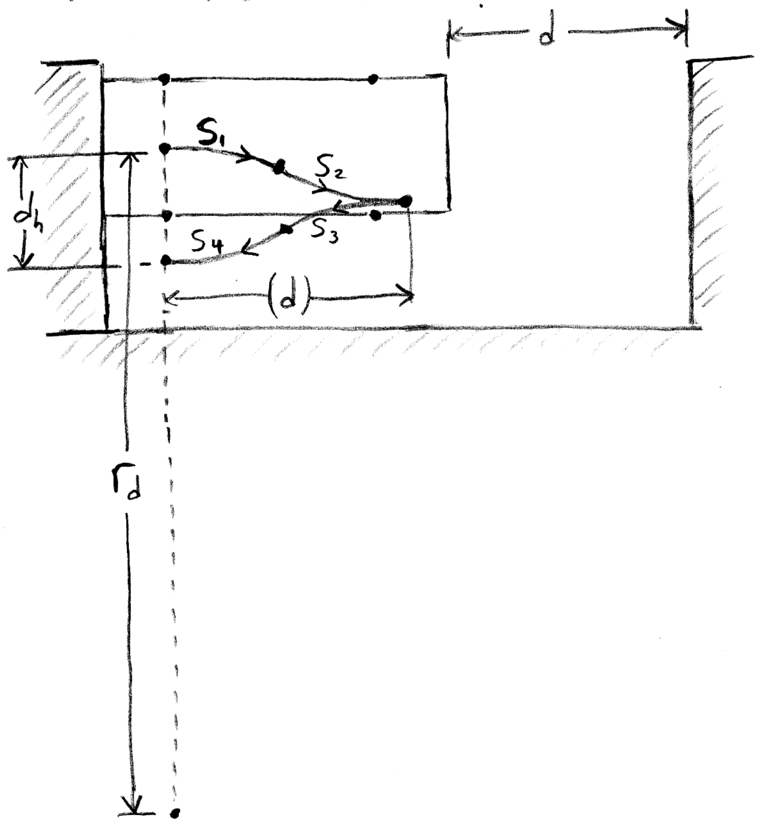
# Parallel Parking Optimization, Pt. 2

Once pulled into a parking spot, parallel to the curb, how much horizontal distance (perpendicular to curb) may be gained by a single forward-reverse motion along optimal arcs of travel?




Parameters $l$, $k$, $j$, $w_o$, and $r$ are defined exactly as they were in Pt. 1.

Similar to Pt. 1:

$d$ = excess length of parking space beyond one full car length. In this case though, $d$ is an independent variable.

$\alpha$ = acute arc angle swept by car for a single arc/center of rotation. Again, this is a dependent variable.

Also:

$r_d$ is defined as distance between center of rotation and the midpoint of the rear axle such that

$$\boxed{r_d \equiv r_b - \frac{w_o}{2} = \sqrt{r^2 - l^2} - \frac{w_o}{2}}$$

**Variable to be solved:**

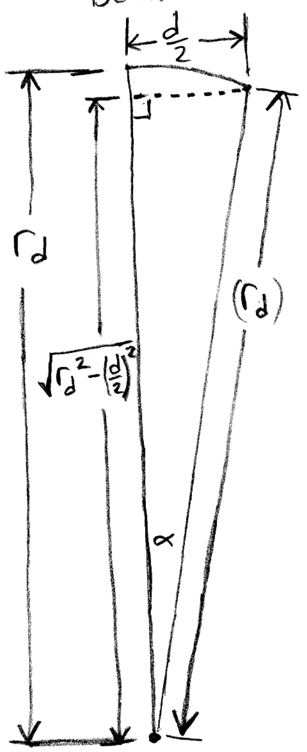
$d_h$ = horizontal displacement (perpendicular to curb) resulting from one full forward-reverse cycle.

## Assumptions:

1. Assumptions 5, 6, & 8 from Pt. 1
2. Optimal gain (maximum $d_h$) is obtained by backing up parallel to curb as far as possible, turning steering wheel hard right (to curb), pulling forward along arc $S_1$, stopping, turning steering wheel hard left (away from curb), pulling forward along arc $S_2$, stopping, turning steering wheel hard right, reversing along arc $S_3$, stopping, turning steering wheel hard left, reversing along arc $S_4$ to end up parallel to curb in such a manner that $S_1 \cong S_2 \cong S_3 \cong S_4$.
3. Car starts displaced far enough from curb such that it doesn't roll up onto curb at any point in the process. Any such contact would most likely occur while traveling along $S_2$, when front right tire is closest to curb.

Derivation of formulas for $\alpha$ and $d$

Detail of single arc, $s_1$




$$\boxed{\alpha = \sin^{-1}\left(\frac{d}{2r_d}\right)}$$

Since $d_h$ includes the displacement of four arcs,

$$d_h = 4\left(r_d - \sqrt{r_d^2 - \left(\frac{d}{2}\right)^2}\right)$$

$$d_h = 4r_d - 2\sqrt{4r_d^2 - d^2}$$

or

$$d_h = 4r_d\left(1 - \sqrt{1-\left(\frac{d}{2r_d}\right)^2}\right) = 4r_d\left(1-\cos\alpha\right)$$

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